

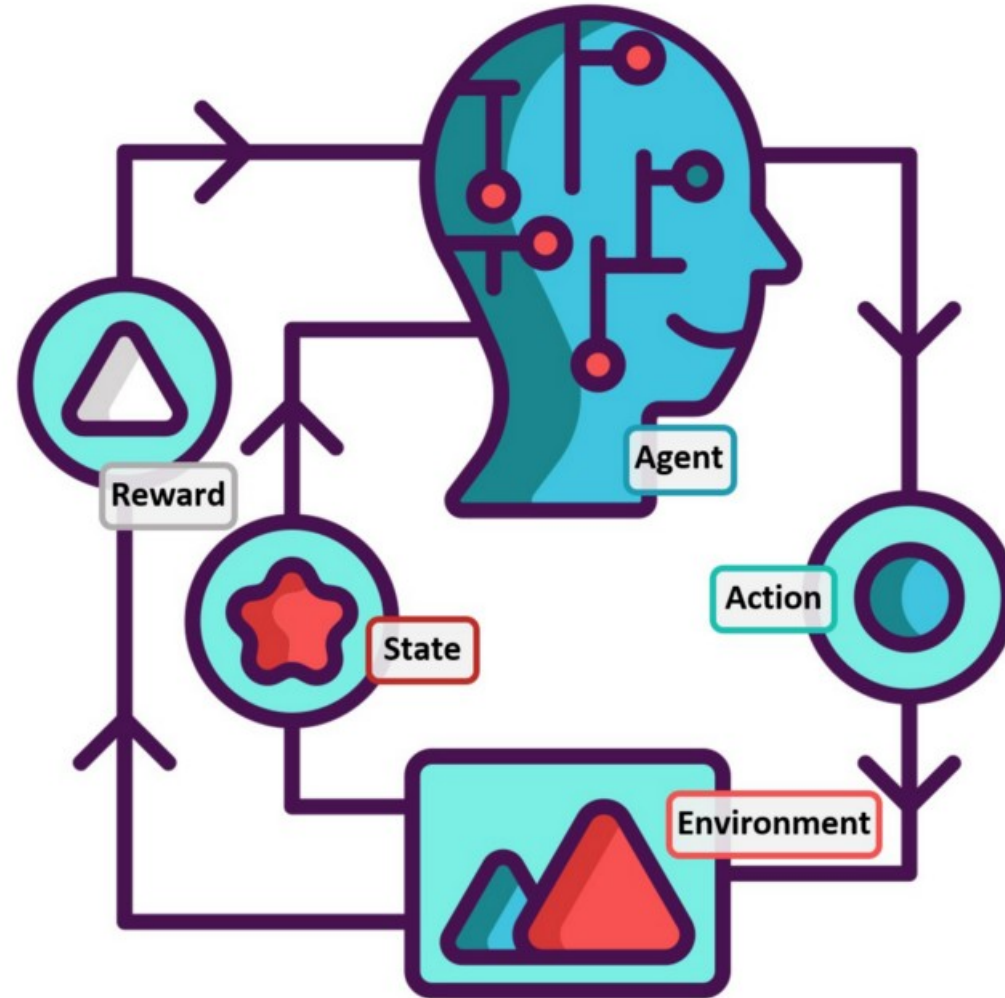


MAFTEC Days  
2023

# Predicate-based explanation of Reinforcement Learning

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Martin C. Cooper – Florence Bannay



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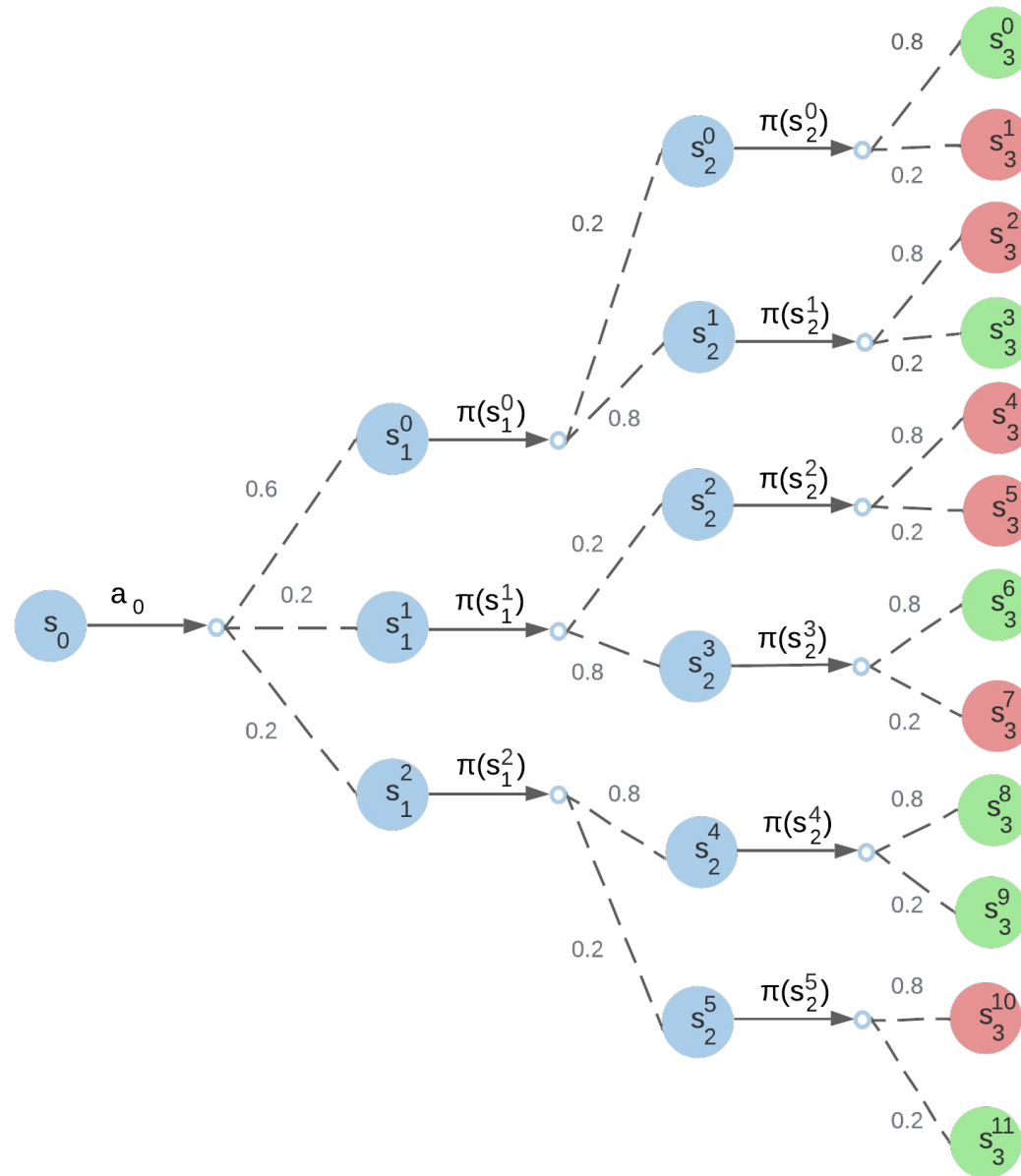
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Action importance score lies in range  $[-1;1]$



-  State
-  predicate  $d$  holds in state
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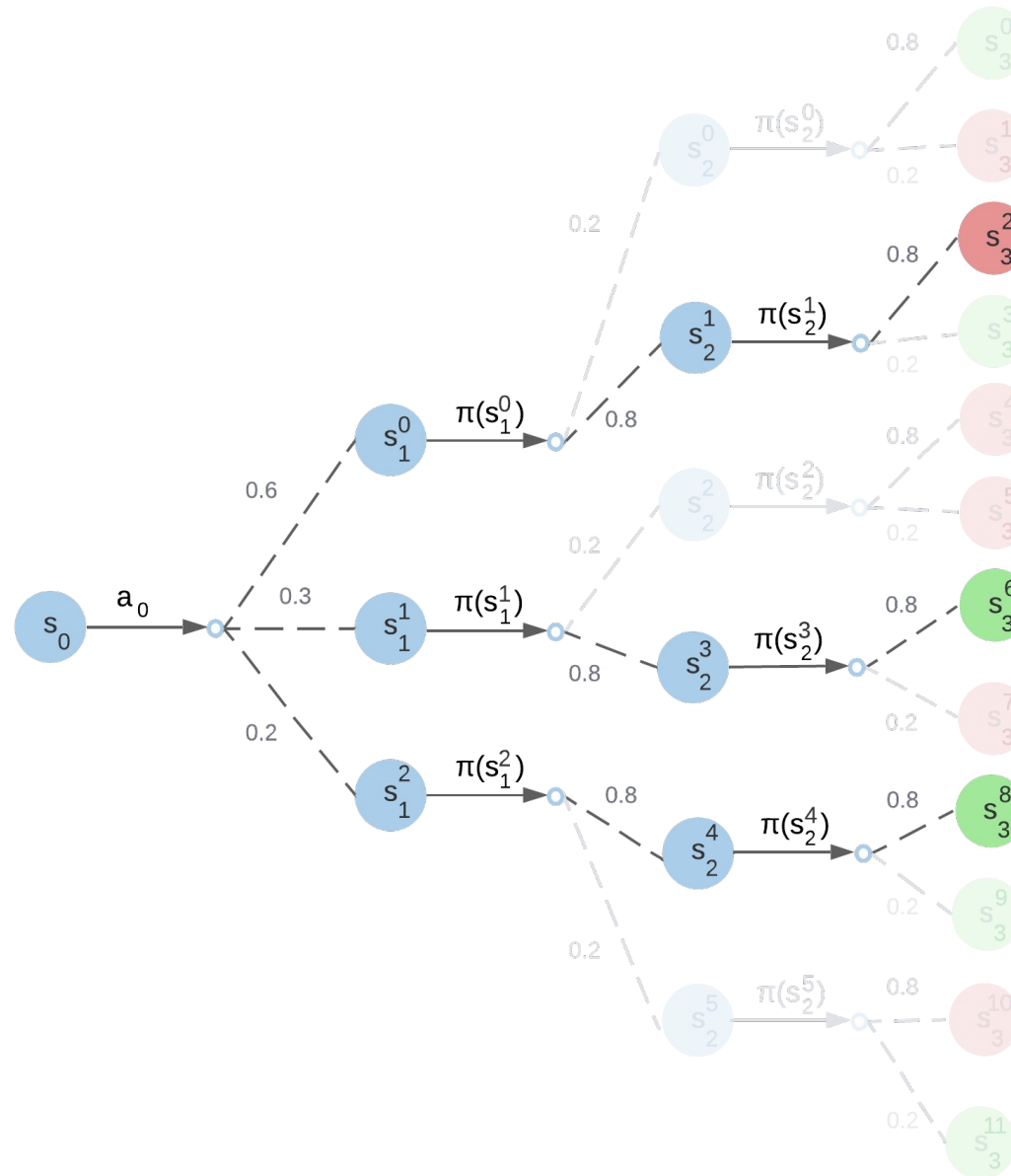
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**Solution:** Generate a large range of scenarios, but not the unlikely ones  
 $n$ -last approximate HXP: most probable transition at the  $n$  last time-step(s)

$n = 2$

Last time-steps



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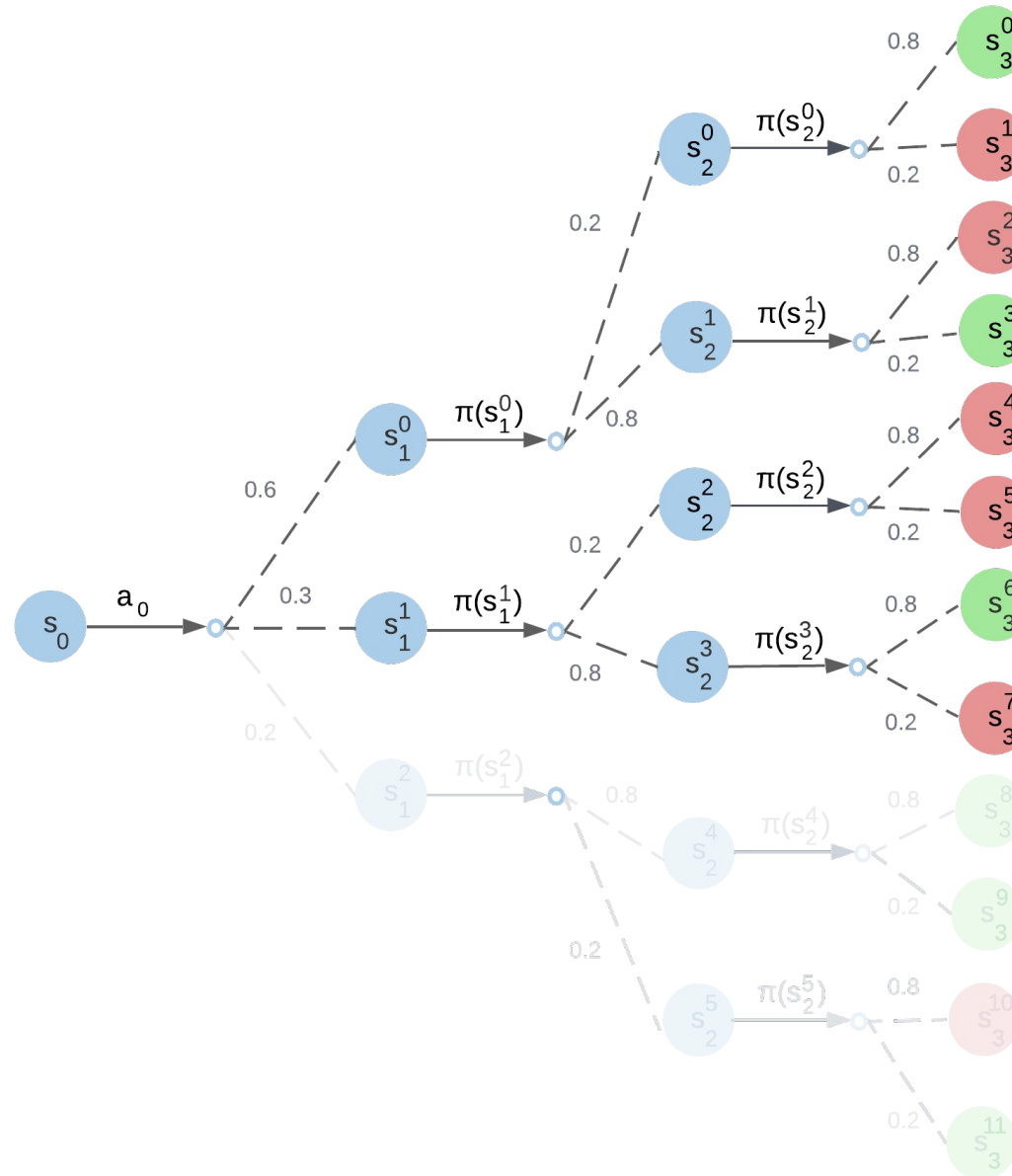
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**Solution:** Generate a large range of scenarios, but not the unlikely ones  
 $m$ -transition approximate HXP:  $m$  most probable transition at each time-step

$m = 2$   
Each time-step



-  State
-  predicate  $d$  holds in state
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-  Action
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**Data:**

- History i.e. state-action sequence  $H = (s_0, a_0, s_1, \dots, a_{k-1}, s_k)$
- Predicate  $d$
- Length of studied sub-sequence  $l$



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Get the associated state  $s_i$

Redefine the predicate to study based on  $s_i$

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Iterate this process through the entire history

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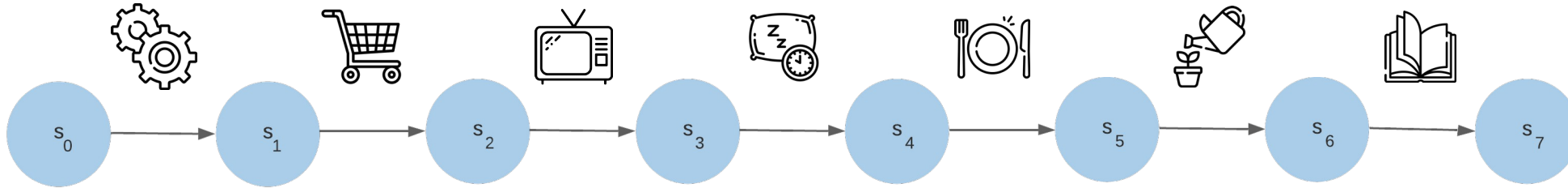
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Iterate this process through the entire history

The re-defined predicate is a general description of a set of states

**Result:** Set of studied predicates and important actions

**Example:** the end of Bob's day



**Bob's state:** (*hunger, happy, tired, fridge, fuel*)

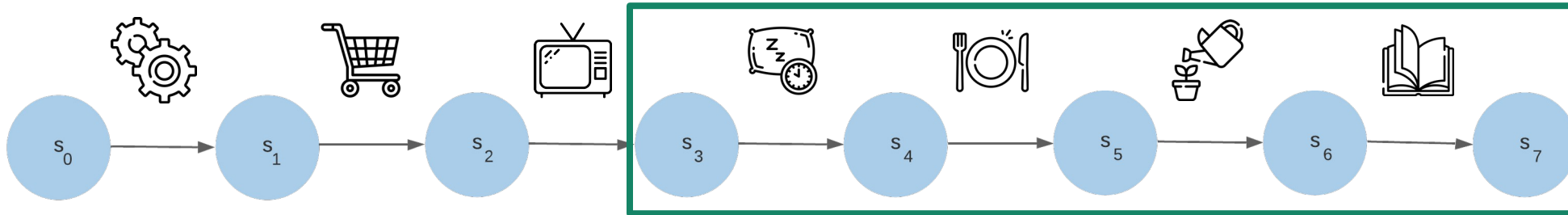
Last history state: ( $\neg$ *hunger, happy, tired,  $\neg$ fridge,  $\neg$ fuel*)

**Data:**

- H: history corresponding to the end of Bob's day
- $d$ : 'Bob is not hungry'
- $l$ : 4

Which actions were important to ensure that  $d$  was achieved, given the agent's policy  $\pi$ ?

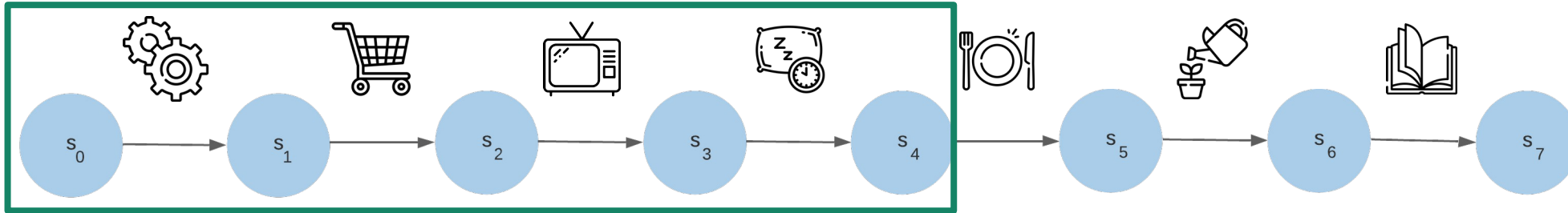
**Example:** the end of Bob's day



Most important action: 'eat'

New predicate based on  $s_4$ ,  $d$ : 'Bob is hungry and has a full fridge'

**Example:** the end of Bob's day



Most important action: *'shop'*

## Result:

- Actions: *'shop', 'eat'*
- Predicates : *'Bob is hungry and has a full fridge', 'Bob is not hungry'*

Bob isn't hungry because he went shopping (to fill his fridge) and then ate

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**Algorithm 2:** Backward HXP algorithm
 

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**Input** :  $H$ : history,  $l$ : maximal sub-sequence length,  $\pi$ : agent's policy,  $d$ : predicate,  $p$ : transition function,  $\delta$ : probability threshold

**Output:**  $A$ : action list,  $D$ : predicate list

$A \leftarrow []$ ;

$D \leftarrow []$ ;

$i_{max} \leftarrow len(H)$ ;

**while**  $i_{min} \neq 0$  **do**

$i_{min} \leftarrow max(0, i_{max} - l)$ ;

$a, s, z, idx \leftarrow select(H_{(i_{min}, i_{max})}, \pi, d, p)$  ; // select a state-action couple

$d \leftarrow all\_PAXp(\mathbb{F}, \kappa, s, \delta, \pi, p, d, i_{max} - i_{min})$  ; // define a new predicate

$A.append(a)$ ;

$D.append(d)$ ;

$i_{max} \leftarrow idx$ ;

**end**

**return**  $A, D$

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# Backward HXP

**State-action couple selection:** Most important action  $a$  and associated state  $s$

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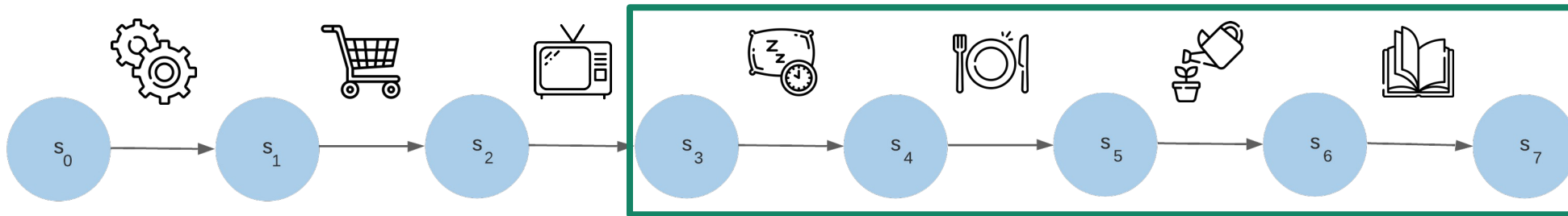
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Weak PAXp: *A subset of fixed features for which the probability of predicting a class  $c$  is at least  $\delta$  (with  $\delta \in [0,1]$ )*

**Example:** Bob's end day



Most important action: 'eat'

Associated state  $s_4$ : (hunger,  $\neg$ happy,  $\neg$ tired, fridge,  $\neg$ fuel)

$u(s_4) = 1, \delta = 0.8$

**Weak PAXp:** (hunger, fridge,  $\neg$ tired)  $\rightarrow$  new predicate

At least 80% of states described by (hunger, fridge,  $\neg$ tired) have a utility of 1

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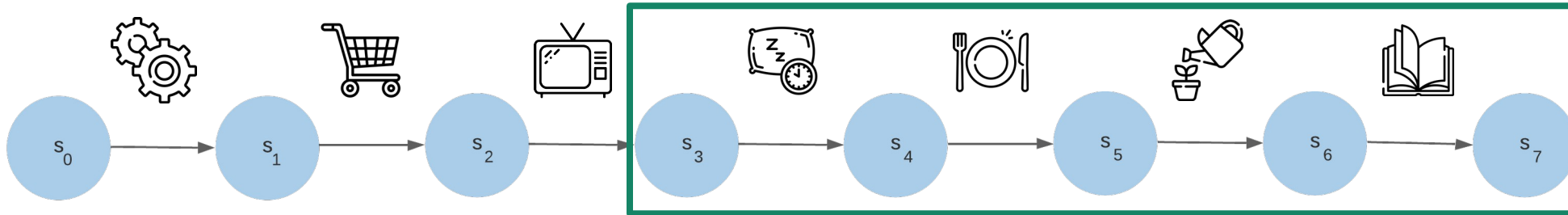
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**Problem:** Finding one *PAXp* is computationnaly expensive

**Solution:** Generate the new predicate with only one *Locally-minimal PAXp*, a class of *Weak PAXp* which is easier to compute



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**Algorithm 2:** findLmPAXp.

---

**Input** : Feature  $\{1, \dots, m\}$ ; feature space  $\mathbb{F}$ , classifier  $\kappa$ , instance  $(\mathbf{v}, c)$ ,  
threshold  $\delta$

**Output:** Locally-minimal PAXp  $\mathcal{S}$

$\mathcal{S} \leftarrow \{1, \dots, m\}$ ;

**for**  $i \in \{1, \dots, m\}$  **do**

**if**  $WeakPAXp(\mathcal{S} \setminus \{i\}; \mathbb{F}, \kappa, \mathbf{v}, c, \delta)$  **then**

$\mathcal{S} \leftarrow \mathcal{S} \setminus \{i\}$ ;

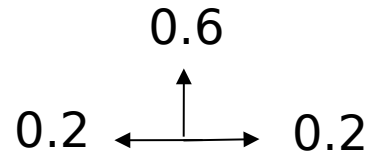
**end**

**end**

**return**  $\mathcal{S}$

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Transition function ( $\uparrow$ )



Actions



State

- Position
- Previous position
- Position of a closest hole
- Distance starting/current position
- Number of holes

Reward function

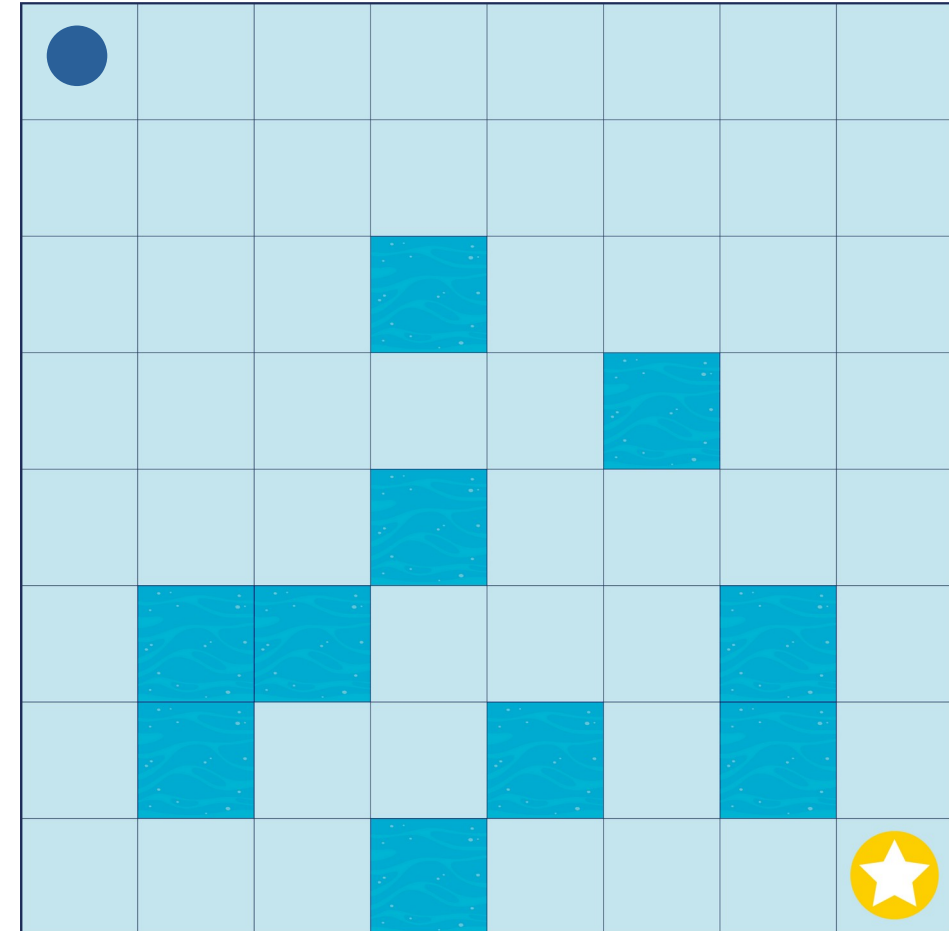
- +1 in Goal position
- +0 otherwise

Algorithm

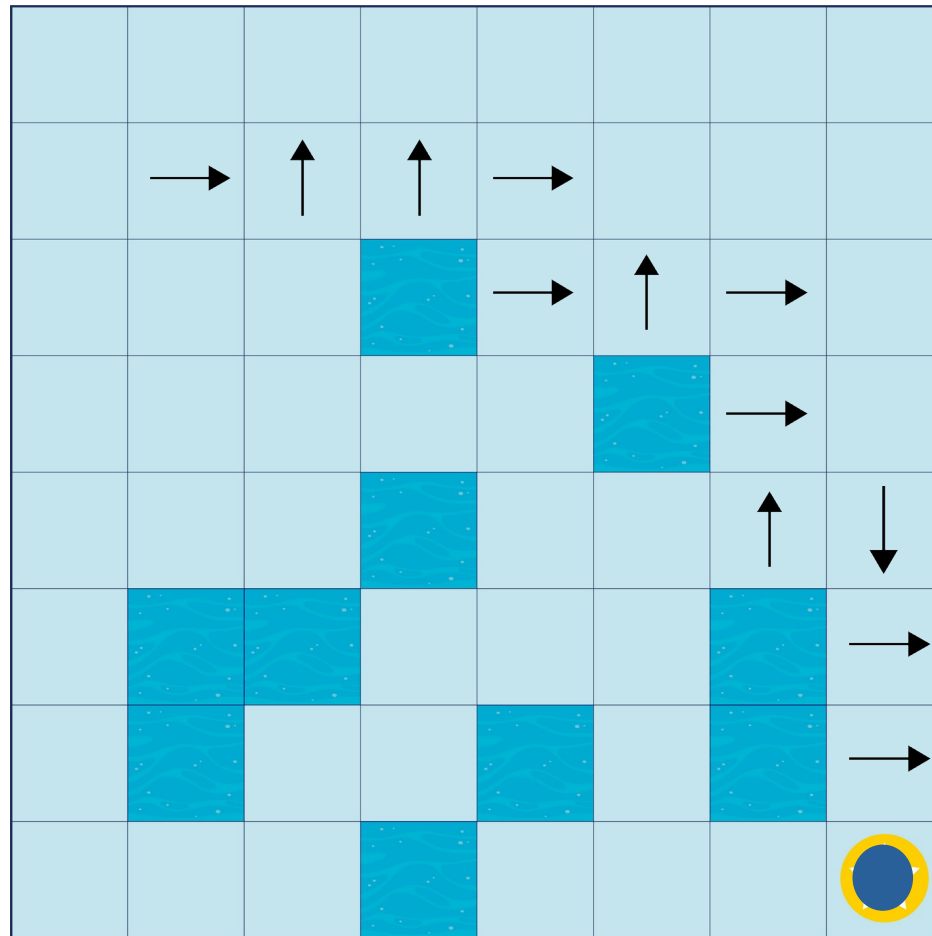
Tabular Q-learning

Predicates

*goal, holes, region*



## History






Predicate: *goal*

B-HXP ( $l = 4, \delta = 0.8$ )

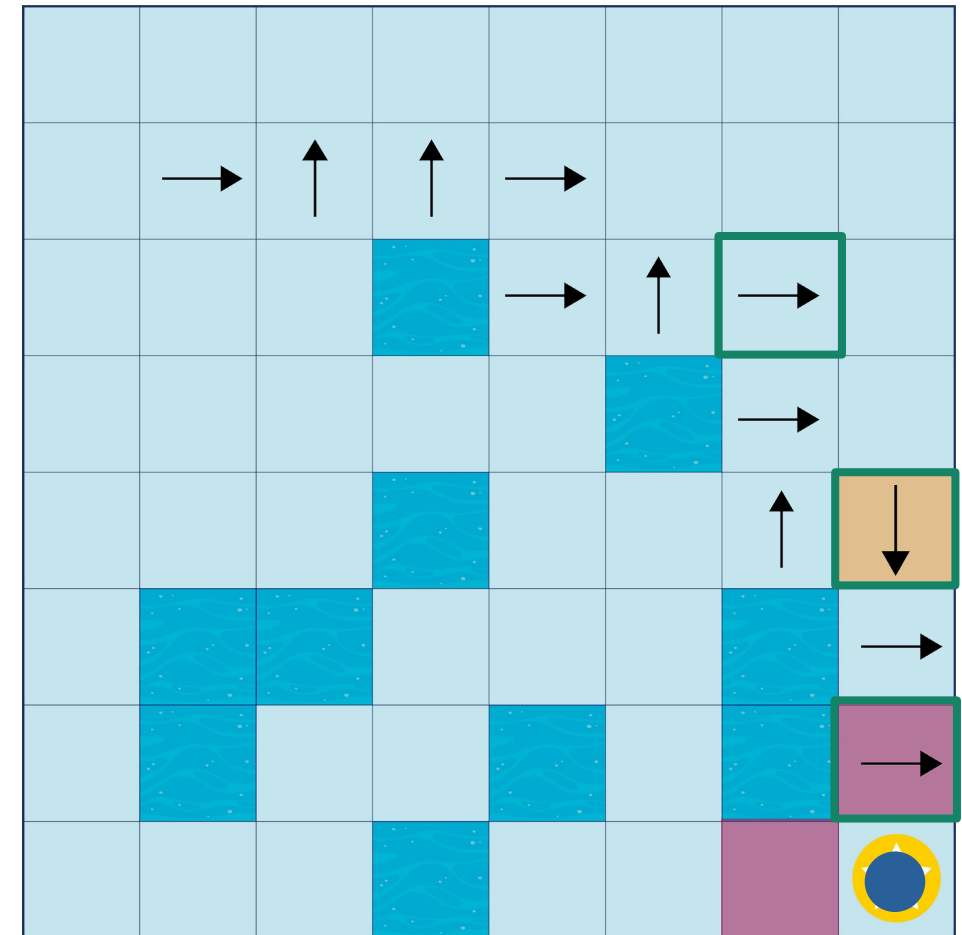
Scores: [-0.0, **0.0**, -0.001, -0.0 |  
 0.006, -0.009, **0.102**, 0.087 |  
 -0.001, 0.04, 0.012, **0.114**]

Predicates:

- Position, Previous position, Close hole position 
- Distance starting/current position 
- Goal 

Runtime: 2.45s

History



Transition function

Obstacles moves

Actions

- Move forward
- Rotate 90° left
- Rotate 90° right

State

7x7 view

Reward function

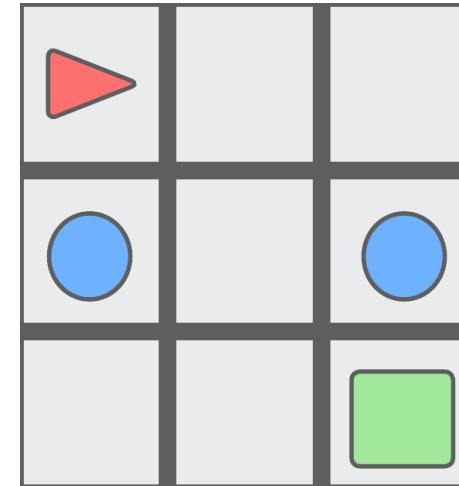
- $1 - 0.9 * t$  if success
- $-1$  if collision

Algorithm

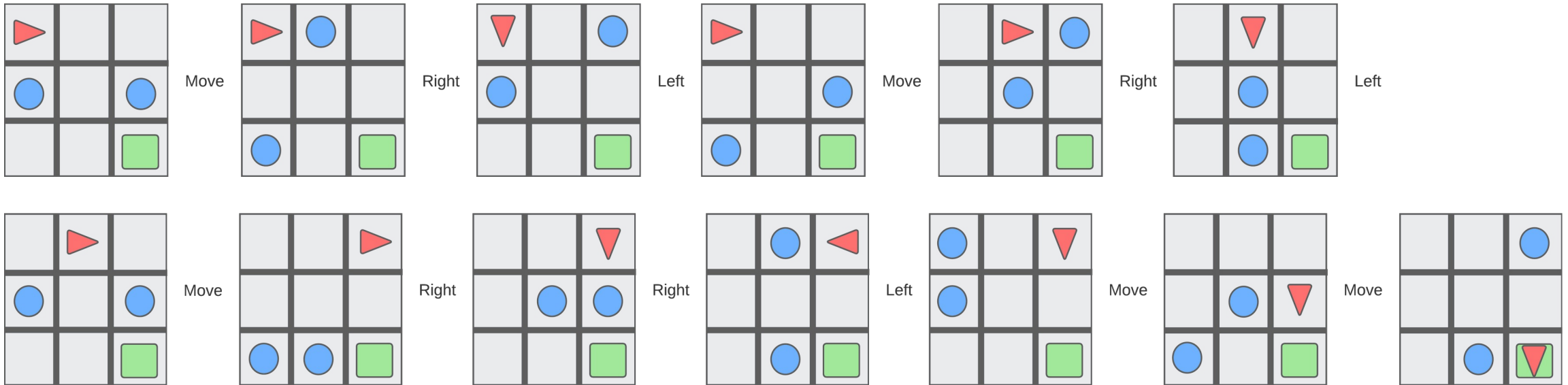
Deep Q Network (DQN)

Predicates

*goal, near obstacles, position*

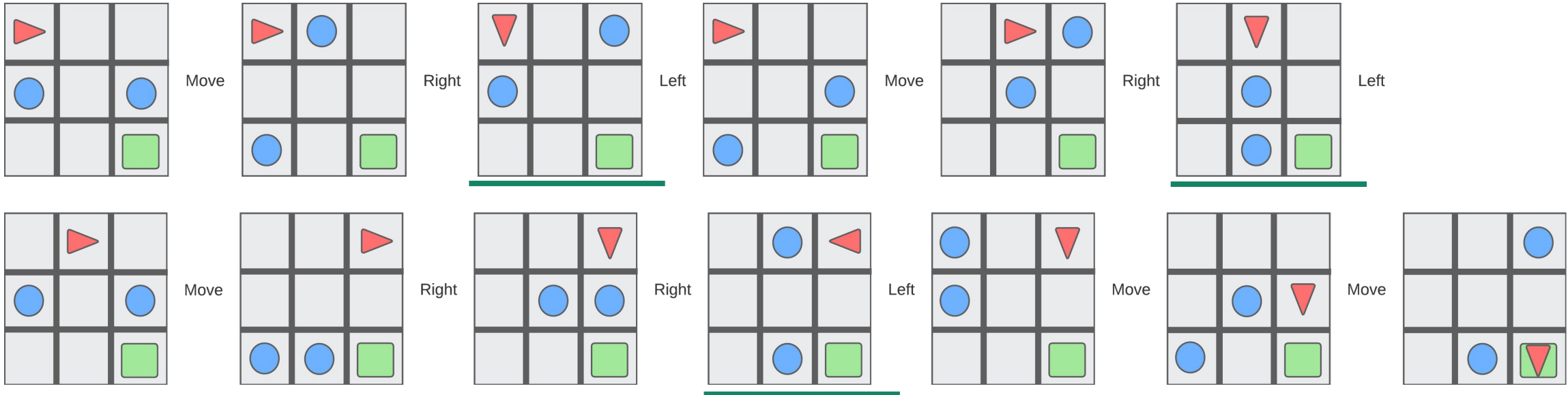


## History



Predicate: *goal*

# Dynamic Obstacles

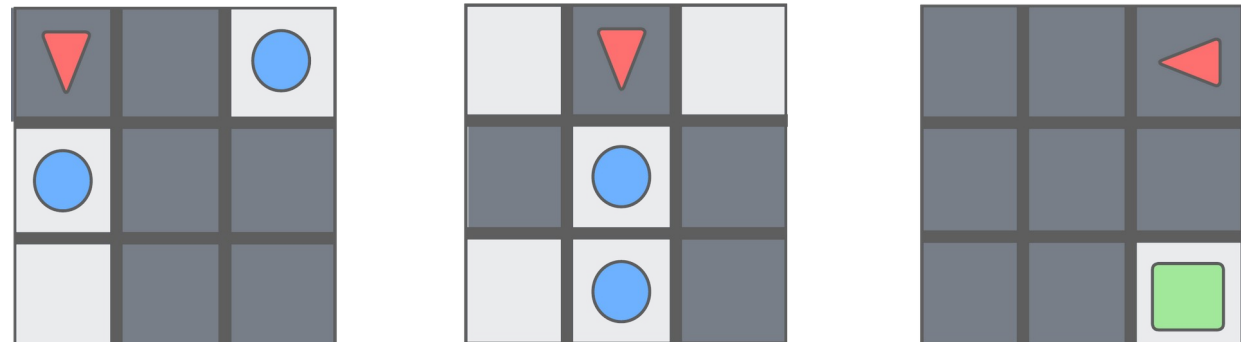


B-HXP ( $l = 4, \delta = 0.9$ )

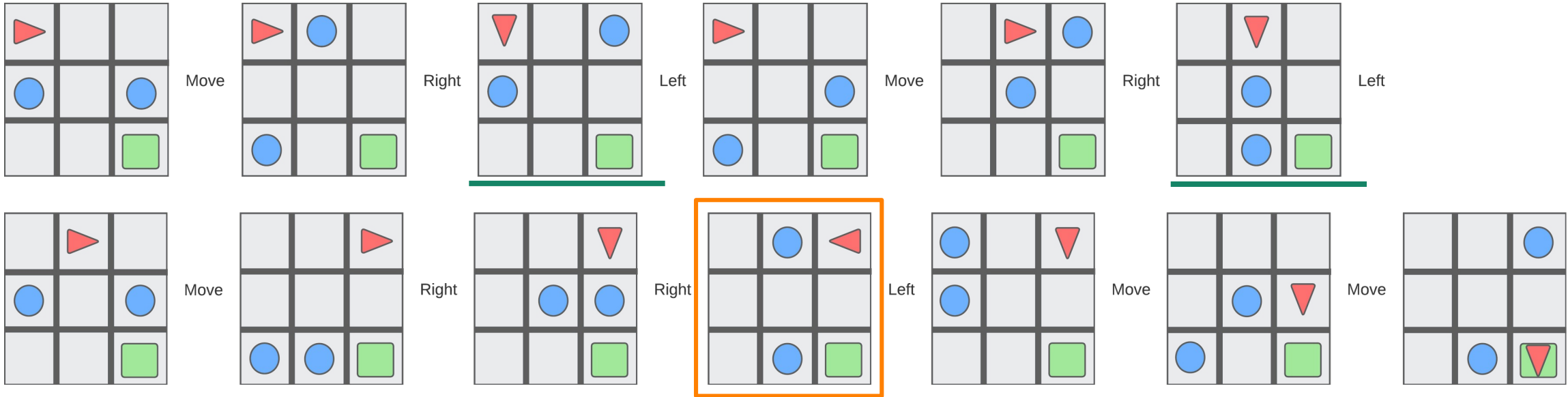
Scores: [-0.009, 0.006 |  
 0.006, **0.012**, -0.004, 0.009 |  
**0.139**, 0.095, -0.029, 0.079 |  
 0.273, **0.48**, 0.42, 0.0]

Runtime: 370s

Predicates:



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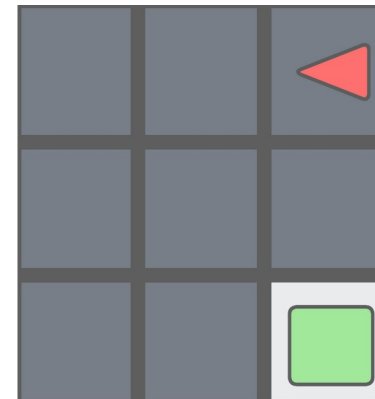


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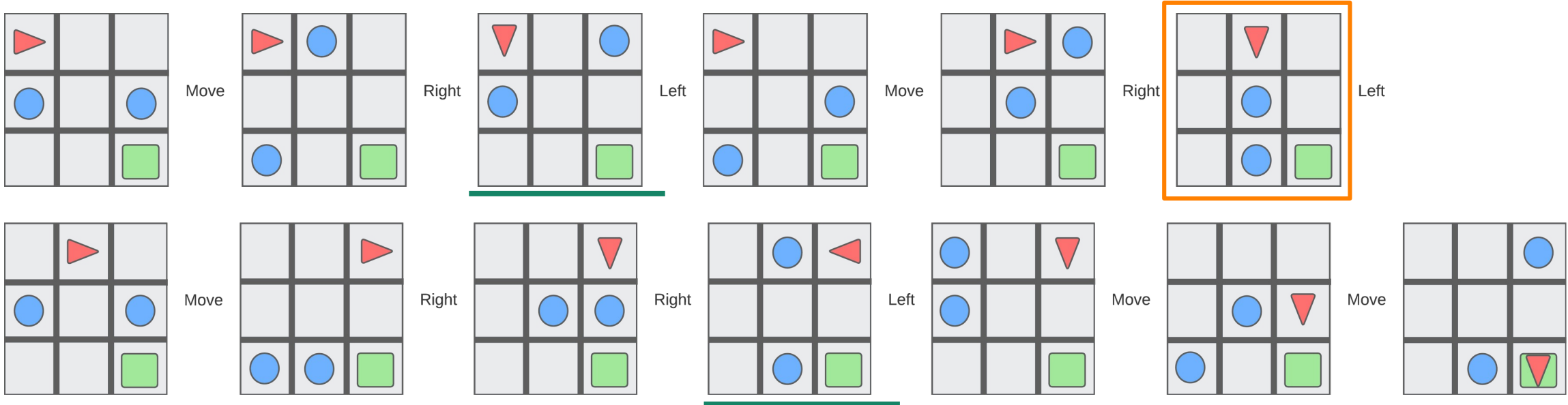
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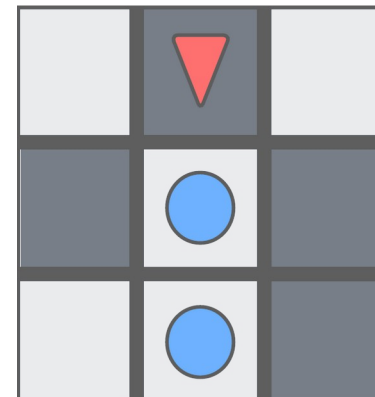


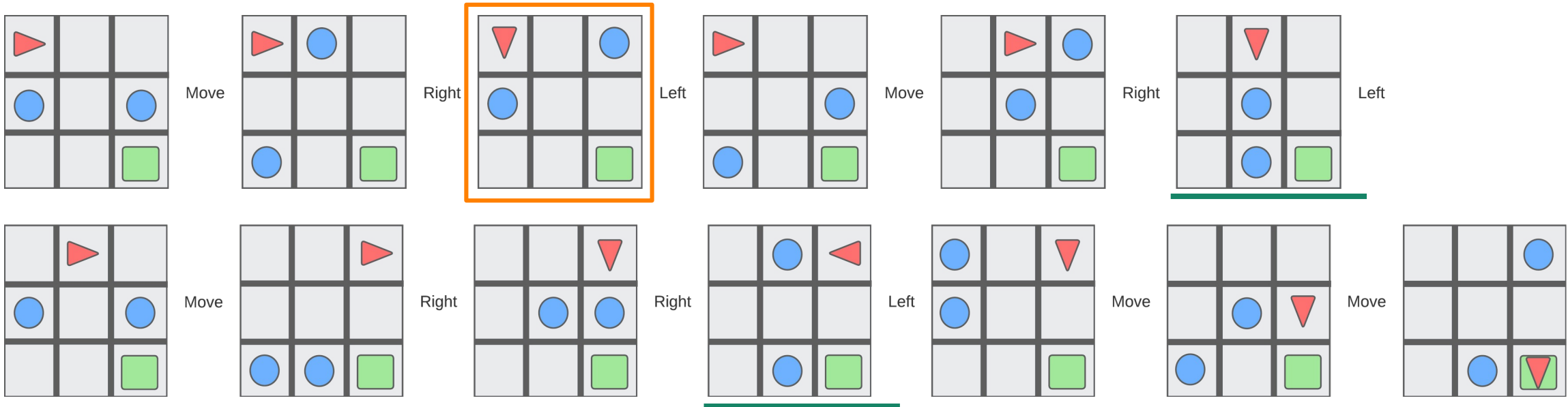
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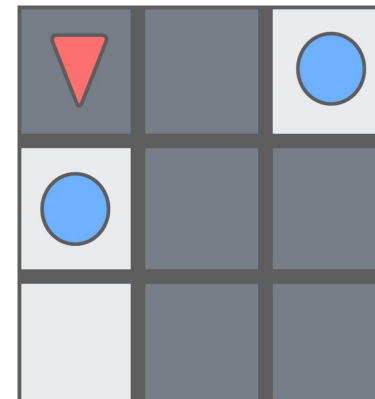


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Predicates:



Transition function      Player 2's policy

Actions                      Column number

State                         Whole board

Reward function

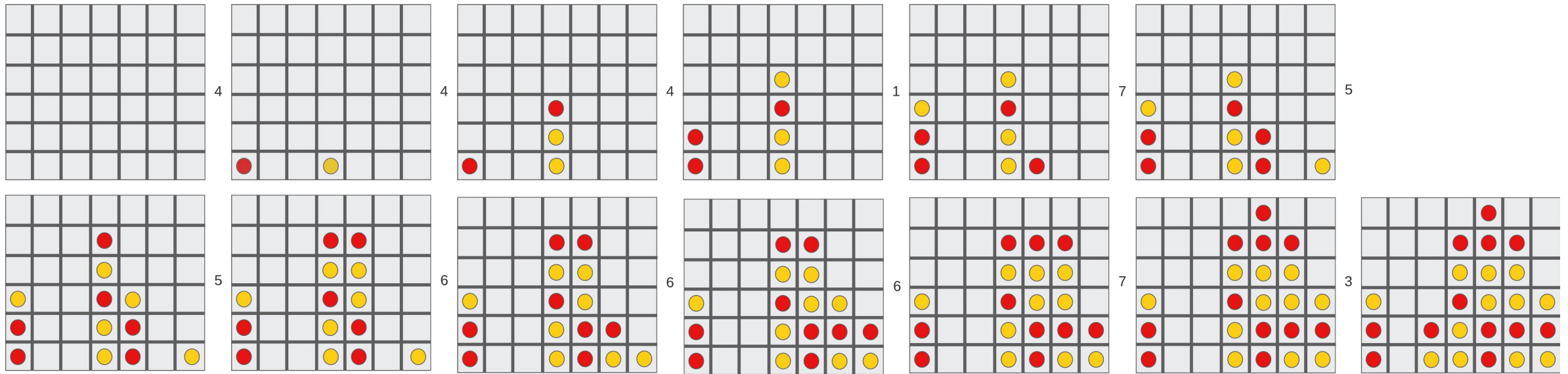
- +1      if win
- -1      if lose
- +0.5   if draw
- +0      otherwise

Algorithm                  Deep Q Network (DQN)

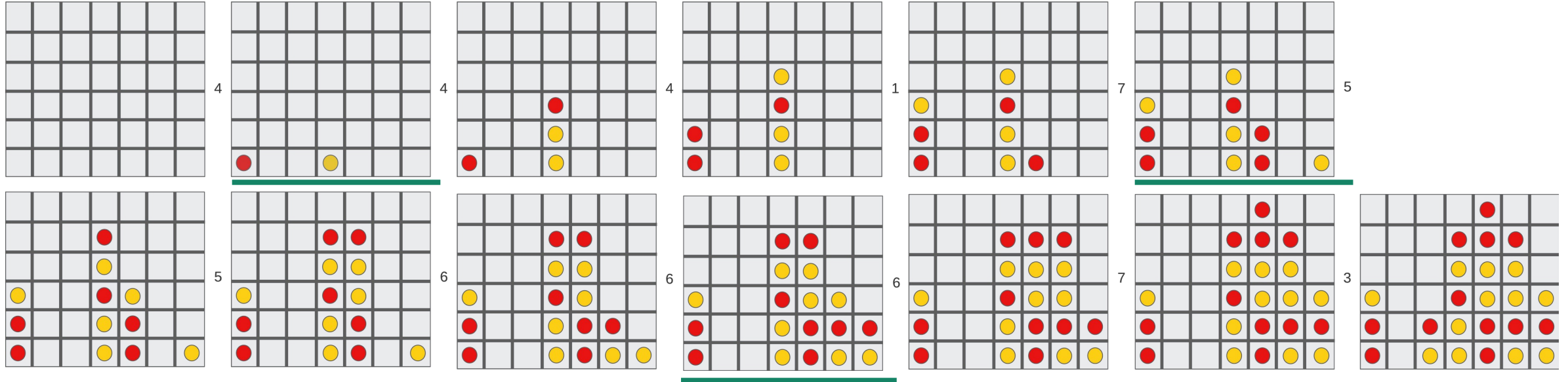
Predicates                *win, lose, 3 in a row, avoid 3 in a row, control mid-column*



## History



Predicate: *win*

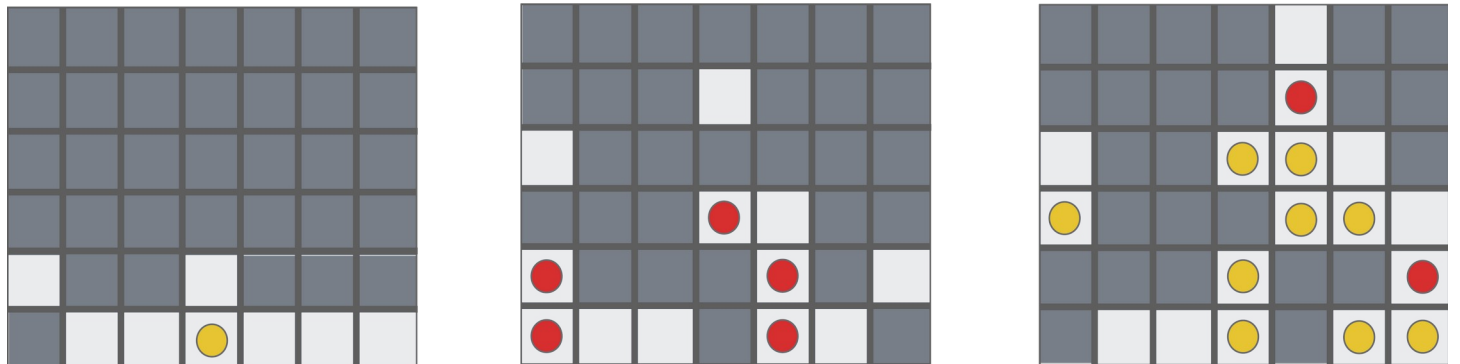


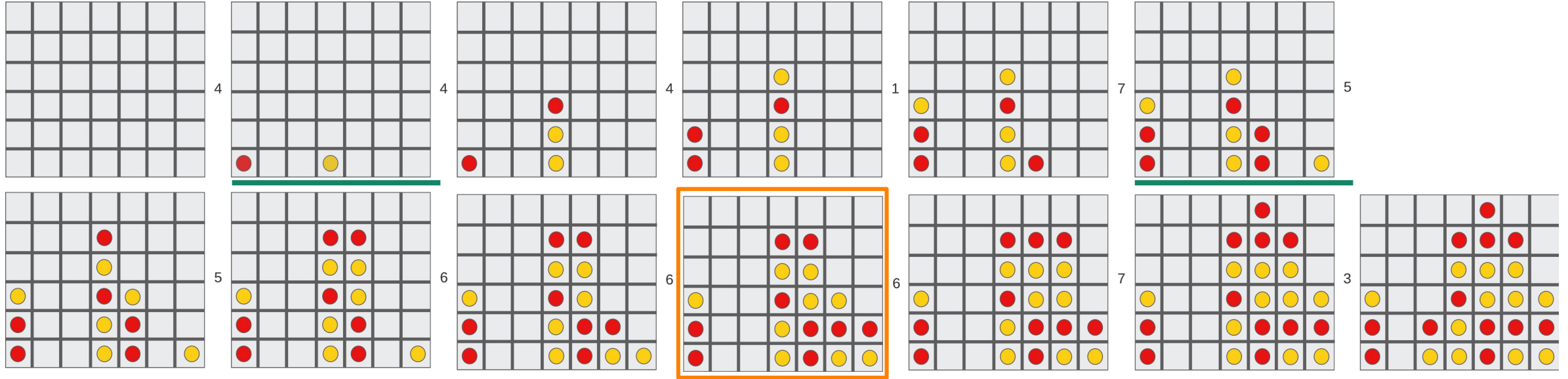
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Runtime: 112s

Predicates:



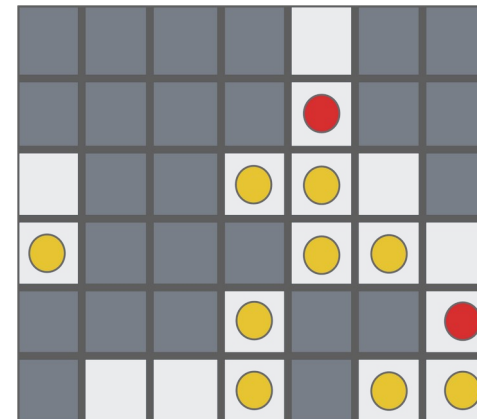


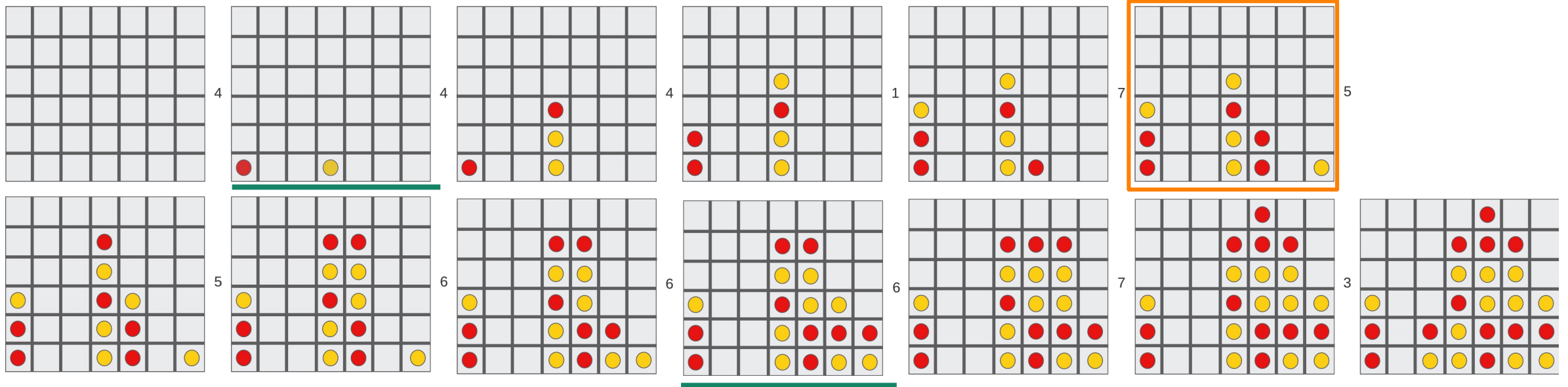
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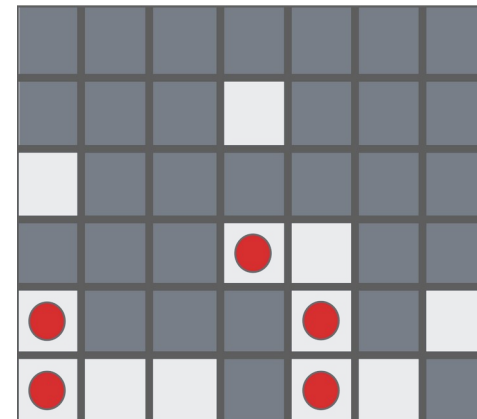


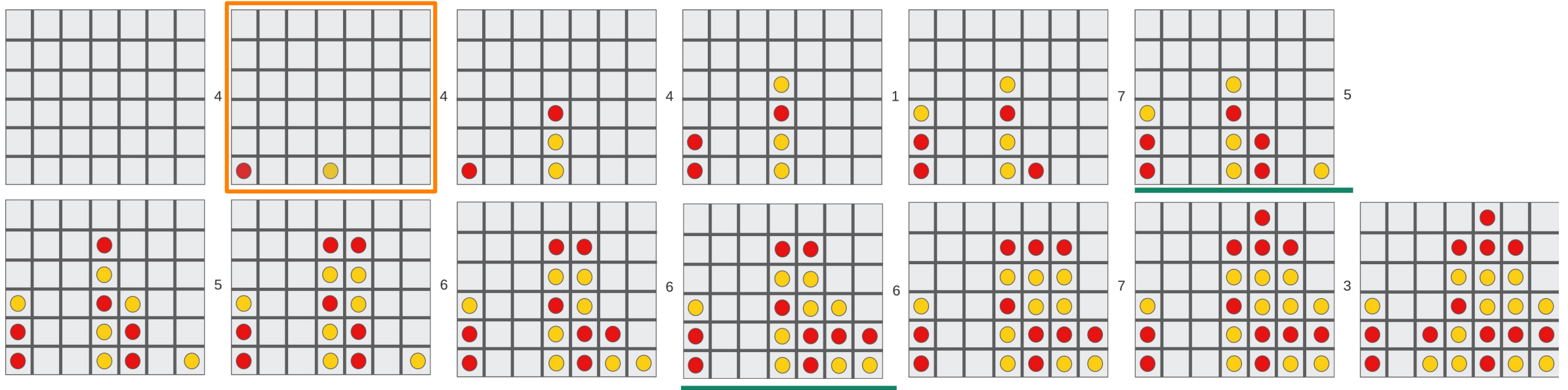
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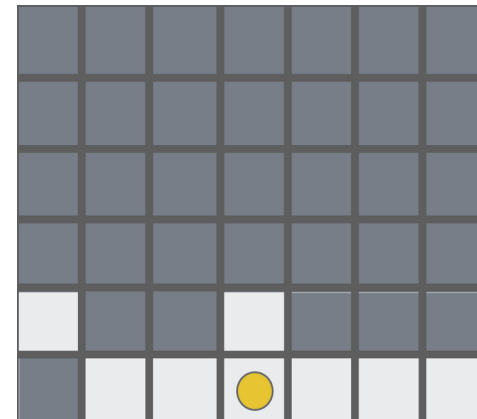


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## HXP:

- Analyse past agent's interactions with the environment:
  - Predicate-based approach
  - Action importance evaluation
- Approximate HXP to reduce computation time

## Backward HXP:

- Analyse past agent's interactions with the environment:
  - Predicate-based approach
  - Action importance evaluation
- Plain HXP
- Approximate computation of PAXp
- Provide to the user important actions and predicates

## Backward HXP:

- Analyse past agent's interactions with the environment:
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  - Action importance evaluation
- Plain HXP
- Approximate computation of PAXp
- Provide to the user important actions and predicates

## Limits:

- Transition function must be known
- Approximate PAXp
- Complexity: importance score and search space for PAXp computation

## Future works:

- Feature ordering heuristics to produce insightful predicates
- Additional experiments